STATS 2MB3, Tutorial 3

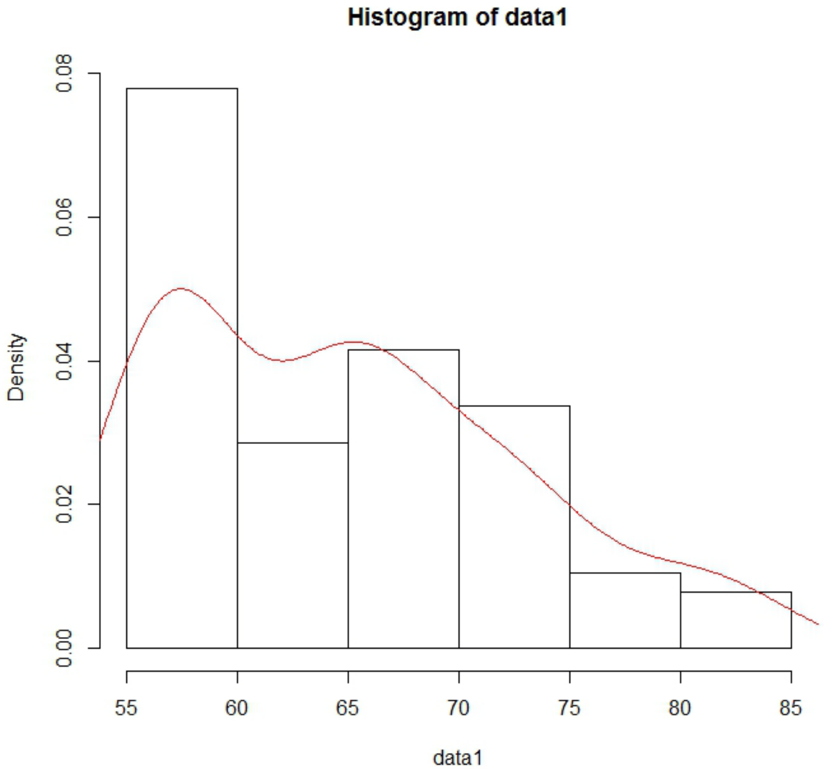
Jan 30th, 2015

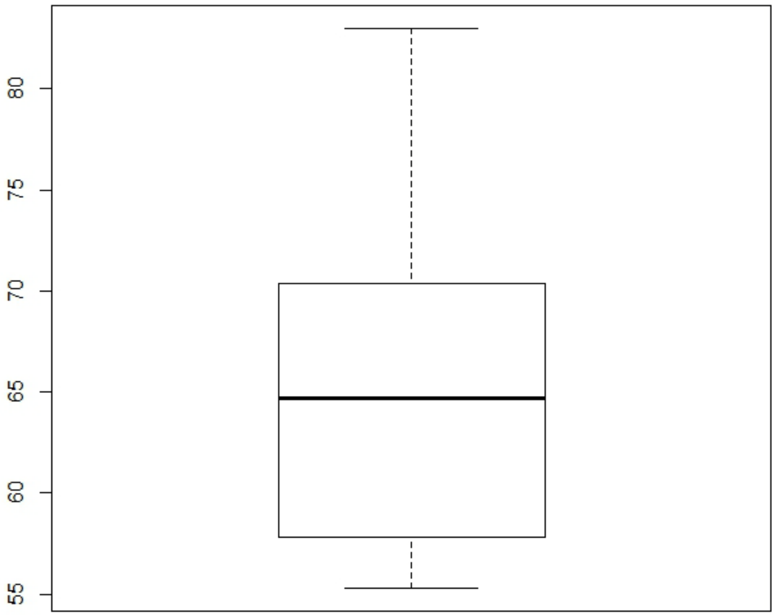
Central Limit Theorem

- Let $X_1, \ldots, X_n$ be a random sample from a distribution with mean μ and variance σ^2. Then if n is sufficiently large, $\overline{X}$ has approximately a normal distribution with mean μ and variance σ^2 / n, and $T_0 = X_1 + \cdots + X_n$ also has approximately a normal distribution with mean $n\mu$ and variance $n\sigma^2$. The larger the value of n, the better the approximation.

Ex 63, page 46

- A sample of 77 individuals working at a particular office was selected and the noise level (dBA) experienced by each individual was determined, yielding the following data:
- 55.3,55.3,55.3,55.9,55.9,55.9,55.9,56.1,56.1,56.1,56.1,56.1, 56.1,56.8,56.8,57.0,57.0,57.0,57.8,57.8,57.8,57.9,57.9,57.9, 58.8,58.8,58.8,59.8,59.8,59.8,62.2,62.2,63.8,63.8,63.8,63.9, 63.9,63.9,64.7,64.7,64.7,65.1,65.1,65.1,65.3,65.3,65.3,65.3, 67.4,67.4,67.4,67.4,68.7,68.7,68.7,68.7,69.0,70.4,70.4,71.2, 71.2,71.2,73.0,73.0,73.1,73.1,74.6,74.6,74.6,74.6,79.3,79.3, 79.3,79.3,83.0,83.0,83.0
- Use various techniques discussed in this chapter to organize, summarize and describe the data.



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The decimal point is 1 digit(s) to the right of the |

5 | 5556666666666777788888899 9
6 | 00022444444
6 | 55555555557777999999
7 | 001113333
7 | 55559999
8 | 333
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Ex 46, page 229

- The inside diameter of randomly selected piston ring is a random variable with mean value 12cm and standard deviation 0.04cm.
- a) If $\overline{X}$ is the sample mean diameter for a random sample of n=16 rings, where is the sampling distribution of $\overline{X}$ centered, and what is the standard deviation of the $\overline{X}$ distribution?
- b) Answer the questions posed in the part (a) for a sample size of n=64 rings.
- c) For which of the two random samples, the one of the part (b), is $\overline{X}$ more likely to be within 0.01cm of 12 cm? Explain your reasoning.

- a)Set X=the inside diameter, then
- E(X)=12, sd(X)=0.04
- $\overline{X}$ is the sample mean of X, then
- $$E(\overline{X}) = E(\frac{X_1 + \cdots + X_{16}}{16}) = \frac{16E(\mathrm{X})}{16} = E(\mathrm{X}) = 12$$
- $$sd(\overline{X}) = \sqrt{Var(\overline{X})} = \sqrt{Var(\frac{X_1 + \cdots + X_{16}}{16})} = \sqrt{\frac{1}{16^2}16 * Var(\mathrm{X})}$$

$$= \frac{1}{\sqrt{16}} * sd(\overline{X}) = 0.04/4 = 0.01$$

- b) When n=64, then

$$E(\overline{X}) = E(\frac{X_1 + \cdots + X_{16}}{64}) = \frac{64E(\mathrm{X})}{64} = E(\mathrm{X}) = 12$$

$$sd(\overline{X}) = \sqrt{Var(\overline{X})} = \sqrt{Var(\frac{X_1 + \cdots + X_{16}}{64})} = \sqrt{\frac{1}{64^2} 64 \cdot Var(\mathrm{X})}$$

$$= \frac{1}{\sqrt{64}} \cdot sd(\overline{X}) = 0.04/8 = 0.005$$

- c)
- $\overline{X}$ in part (b) is more likely to be within 0.01cm of 12cm, because part (b) has a smaller standard deviation.

Ex 81, page 236

- If the sample size N is also a random number and it is independent of the observations X, it can be shown that

$$E(X_1 + \cdots + X_N) = \mathrm{E(N)} \cdot \mu$$

- a) If N=the number of components that are brought into a repair shop on a particular day, and X_i denotes the repair time for ith component, we know the expected number of components is 10 and expected repair time for each component is 40 min, then what is the expected total repair time?

- $E(N) = 10$ and $\mu = 40$, then the expected total repair time is 400.